**Grade 5 Mathematics, Quarter 3, Unit 3.1**

# Understanding and Measuring Volume Using Manipulatives

## Overview

**Number of instructional days:** 12 (1 day = 45–60 minutes)

### Content to be learned

- Define volume as an attribute of a solid figure.
- Comprehend the concept of volume measurement.
- Comprehend the concept of unit cube
- Measure volume by counting in cubic cm., cubic in., cubic ft., and improvised units.
- Relate the number of unit cubes to the volume of a solid figure.

### Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Use varied representations and approaches when solving problems.

Model with mathematics.

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Analyze mathematical relationships to draw conclusions.

### Essential questions

- What is volume?
- How is volume an attribute of a solid figure?
- How do you use unit cubes to measure volume?
- What are the attributes of a cube?
- How can you measure volume?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| **Measurement and Data** | **5.MD** |
|---|---|

**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

## Common Core Standards for Mathematical Practice

**2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## Clarifying the Standards

*Prior Learning*

In grade 3, students realize that a square with a side length of 1 unit is called a “unit square” and has “one square unit” of area. They use it to measure area by covering a plane figure completely with no gaps to show area in square units. They measure the square units by counting.

In grade 4, students use formulas for area and perimeter of rectangles to solve real world problems, like finding the width of a rectangular room when given the area of the rug and the length.

*Current Learning*

In grade 5, students will realize that a cube with a side length of 1 unit, called a unit cube, has one cubic unit of volume. This is both at the reinforcement and developmental levels because students know about unit cubes previously, but now they learn it has a volume of 1 cubic unit. Students also learn that when they pack a solid figure without gaps using an amount (n) of cubes, then the volume is (n) cubic units. Students can count the cubes to determine the volume. Students already counted squares in third grade to find area. So counting cubes is at the reinforcement level, and finding volume is reinforcement because it builds on area. This content is a critical level of instruction.

*Future Learning*

In grade 6, students will extend their understanding to find volume of right rectangular prisms with fractional edge lengths using unit cubes. They will be able to show it is the same as multiplying lengths.

## Additional Findings

Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. (Curriculum *Focal Points,* NCTM, p. 17)

### Grade 5 Mathematics, Quarter 3, Unit 3.2

# Linking Concrete Measurement of Volume to the Use of Formulas

## Overview

**Number of instructional days:** 12 (1 day = 45–60 minutes)

### Content to be learned

- Relate volume to the operations of multiplication and addition.
- Determine that the volume of a right rectangular prism can be found by building and counting units in a solid.
- Determine that the volume of a right rectangular can be found by multiplying LxWxH.
- Determine the volume of a right rectangular prism can be found by multiplying the height by the area of the base.
- Understand the relationship of how the three different methods of determining volume are related to the actual representation.
- Solve real world and mathematical problems involving volume.

### Mathematical practices to be integrated

Use appropriate tools strategically.

- Use tools when solving a mathematical problem and to deepen their understanding of concepts.
- Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.

Look for and express regularity in repeated reasoning.

- Look for mathematically sound shortcuts.
- Use repeated applications to generalize properties.

### Essential questions

- How can you find the volume of a right rectangular prism with unit cubes?
- How can you use formulas to find the volume of a right rectangular prism?
- What is the relationship between volume formulas and the use of unit cubes?
- How does the volume formulas represent the associative property of multiplication?
- How can the volume formulas be used to solve real world problems?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| Measurement and Data | 5.MD |
|---|---|

**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

## Common Core Standards for Mathematical Practice

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Clarifying the Standards

*Prior Learning*

In grade 3, students find the area of a rectangle with whole number side lengths by tiling it. This proves that the area is equivalent to multiplying side lengths.

*Current Learning*

In grade 5, students will pack a rectangular prism with unit cubes and demonstrate that the volume is the same as counting cubes as multiplying the edge lengths and by multiplying the height by the base. Students will also use the formulas for volume (v=LxWxH and V=BxH) for rectangular prisms with whole number lengths. Students will be solving both real world and mathematical problems involving volume. Grade 5 students will also start to add the volume of two different solid figures together. Students have found area and perimeter before with formulas, comparing them to counting manipulatives. This is the first time they are finding volume with a formula. They have used formulas with plane and 2-D figures. Now solid figures are introduced. So, it is reinforcement level, but it is a critical content area. Also, students have previously added areas before and now they are adding volume. This is at the reinforcement level of development.

*Future Learning*

Grade 6 students will count unit cubes after packing right rectangular prisms with fractional edge length measures and demonstrate the equivalence to the use of formulas (v=L x W x H and v= B x H) by multiplying the edges of the prism.

## Additional Findings

Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. (*Curriculum Focal Points,* NCTM, p. 17)

Grade 5 Mathematics, Quarter 3, Unit 3.3

# Solving Real-World Problems Involving Volume

## Overview

**Number of instructional days:** 10 (1 day = 45–60 minutes)

### Content to be learned

- Recognize finding the volume of solid figures that are made up of two non-overlapping right rectangular prisms as an additive property.
- Determine the volume of solid figures that are made up of two non-overlapping right rectangular prisms.
- Solve real world problems involving volume by applying the technique of adding the volumes of the non-overlapping parts.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Identify and execute appropriate strategies to solve the problem.
- Check their answers using a different method and continually ask, "Does this make sense?"

Construct viable arguments and critique the reasoning of others.

- Justify their conclusions, communicate them to others, and respond to the arguments of others.
- Understand and use prior learning and their relationships in problem situations.

### Essential questions

- How can you use unit cubes to determine the volume of a rectangular prism?
- How can you use formulas to determine the volume of a rectangular prism?
- Why would knowledge of volume be important to a person in everyday life?
- How many different models of a rectangular prism that each have a volume of 24 cubic units can you create, how do you know?
- How would you find the total volume of a building that is composed of two different rectangular prisms?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| **Measurement and Data** | **5.MD** |
|---|---|

**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

## Common Core Standards for Mathematical Practice

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## Clarifying the Standards

*Prior Learning*

Grade 4 students utilized area and perimeter formulas for rectangles in real world context and problem solving situations.

*Current Learning*

In grade 5, students utilize formulas for volume (V = L x W x H and V = B x H) within the framework of real world problem solving to find volumes of rectangular prisms and right rectangular prisms with whole number edges.  Students demonstrate the ability to add volumes in a real world context.  This is a critical content area that builds on grade three and fours work with area and perimeter and prior work in these grades relating to content to real world problem solving.  This makes this reinforcement level of learning since it is building upon prior learning.

*Future Learning*

In grade 6, students will utilize the formulas for volume of rectangular prisms with fractional edge lengths within a real world problem solving framework.

## Additional Findings

Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. (*Curriculum Focal Points,* NCTM, p. 17)

**Grade 5 Mathematics, Quarter 3, Unit 3.4**

# Using Measurement Conversions

## Overview

**Number of instructional days:** 6 (1 day = 45–60 minutes)

### Content to be learned

- Convert among different sized units within a given measurement system.
- Use conversions to solve multistep real-world problems.

### Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Flow between contextual and non-contextual situations during problem solving and make meaning of numbers and symbols.
- Consider units involved.

Use appropriate tools strategically.

- Use grade-level appropriate tools available.
- Decide when tools are appropriate and helpful.

Attend to precision.

- Define mathematical symbols and units of measure consistently and appropriately.
- Calculate accurately by rechecking for precision.

Look for and make use of structure.

- Look for patterns to simplify.

### Essential questions

- How do you convert one unit of measure to another?
- What patterns do you notice when converting from one measurement to another?
- How can measurement conversion be applied to real-life problems?
- How can the relationship between the units of measurements being converted be explained?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| Measurement and Data | 5.MD |
|---|---|

**Convert like measurement units within a given measurement system.**

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

## Common Core Standards for Mathematical Practice

**2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

## Clarifying the Standards

*Prior Learning*

By the end of fourth grade, students knew the relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, students expressed measurements in a larger unit in terms of a smaller unit. They used all four operations to solve problems using a variety of representations of measurements. They also used diagrams and number lines to represent quantities. Fourth-grade students used real-world situations (tiling floor, fence around a garden) to find area and perimeter.

*Current Learning*

Students convert among different-sized standard measurement units within a given measurement system. (e.g., convert 5 cm to 0.05 m) Use these conversions in solving multistep, real-world problems.

*Future Learning*

Students in grade 6 will be expected to use ratio reasoning to convert measurement units. They will manipulate and transform units appropriately when multiplying or dividing quantities. Sixth-grade students solve real-world and mathematical problems by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

## Additional Findings

According to *Principles and Standards for School Mathematics*, “students should gain facility in expressing measurement in equivalent forms. They use their knowledge of relationships between unit and their understanding of multiplicative situations to make conversions, such as expressing 150 cm as 1.5 meters or 3 feet as 36 inches. Since students in the United States encounter two systems of measurement, they should also have convenient referents for comparing units in different systems—for example, 2 cm is a little less than an inch, a quart is a little less than a liter, a kilogram is about two pounds. However, they do not need to make formal conversions between the two systems at this level” (p. 172).

According to the *PARCC Model Content Frameworks*:

“Conversions within the metric system represent an important practical application of the place value system. Students’ work with these units (5.MD.1) can be connected to their work with place value” (5.NBT.1) (p. 24).

“When students explain patterns in the number of zeros of the product when multiplying a number by powers of ten (5.NBT.2), they have an opportunity to look for and express regularity in repeated reasoning (MP.8). When they use these patterns in division, they are making sense of problems (MP.1) and reasoning abstractly and quantitatively” (MP.2) (p. 25).